SOLUTIONS OF BOREHOLES PROBLEM BY ANALYTICAL METHOD

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l.O Introduction

In mining surveying, exploratory drill holes are driven in search of new deposit. To optimize the mining operation, it is of a prime interest to know the nature of the surface of the ore body.

In the case of a plane surface, the most important parameters are the azimuth of dip, azimuth of strike and angle of dip (inclination) of the plane. These parameters are easily determined when the equation of the plane is known. From then on, the boreholes problem can be solved easily using simple trigonometrical manipulations.

In the past, the above mentioned parameters are determined semi-graphically. The computation varies from one case to another and sometimes involves complicated diagrams which therefore restricts the use of computers. This paper attempts to develop an analytical method to determine those parameters so that high speed computers can be used. Examples will be shown to clarify the problem.

2.0 Determination of the Equation of Plane

When there is a number of boreholes (>3) driven to the ore body and their coordinates (x, y, z) are known, the equation of the plane of the ore body can be determined using least squares method. The general equation of the plane is given as (Kreyszig, E, 1979):

$$- D = O \qquad - \quad (1.1)$$

where *A*, B, C and D are the coeffcients.

Dividing equation (1.1) by C' gives

$$S_1 X + S_2 Y + Z + C_o = O \qquad - (1.2)$$

where

$$S_1 = \frac{A}{C'}; \quad S_2 = \frac{B}{C'}; \quad C_o = \frac{D}{C'} \qquad - (1.3)$$

and X, Y, Z are coordinates of points.

Equation (1.1) is reduced to (1.2) to make parametric adjustment possible by assuming that X and Y as errorless leaving Z as the only observable. The assumption is based upon the fact that X and Y are more accurately determined compared to Z due to the nature of the measurement as well as instrumentation. Remembering that in mining surveying depths are measured, equation (1.2) is written in the form of

$$S_1 X + S_2 Y - Z + C_o = O \qquad - (1.3)$$

Using equation (1.3) as the mathematical model to determine the equation of the plane, observation equations for n points can then be formed as follows:

$$\begin{aligned} S_1 X_1 + S_2 Y_1 - Z_1 + C_o - v_1 &= O \\ S_1 X_2 + S_2 Y_2 - Z_2 + C_o - v_2 &= O \\ &\vdots \\ S_1 X_n + S_2 Y_n - Z_n + C_o - v_n &= O \end{aligned} \qquad - (1.4)$$

As we can see there are three unknowns to be determined, S_1, S_2 and C_o. These unknowns are then determined by Least Squares Parametric Adjustment Method of the model (Wells, D.E. and Krakiwsky, E.J., 1971:

$$F(\bar{X}) = \bar{L} \qquad - (1.5)$$

where $\bar{X}$ and $\bar{L}$ are the unknown and observable vectors respectively Linearisation of the above model gives

$$F(\bar{X}) - \bar{L} = F(X^{o}) + \left.\frac{\partial F}{\partial \bar{X}}\right|_{X^{o}} X - (L + V) = O \qquad \text{- (1.5a)}$$

or

$$W + AX - V = O$$

where $W = F(X^{o}) - L$ and $A = \left.\frac{\partial F}{\partial \bar{X}}\right|_{X^{o}}$

and

$$A = \begin{bmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ \vdots & \vdots & \vdots \\ X_n & Y_n & 1 \end{bmatrix} \qquad \text{- (1.6)}$$

$B = \pm I$ - (assumed)

$$W = f_1(X^{o}, L) = \begin{bmatrix} S_1^{o} X_1 + S_2^{o} Y_1 - Z_1 + C_o^{o} \\ S_1^{o} X_2 + S_2^{o} Y_2 - Z_2 + C_o^{o} \\ \vdots \\ S_1^{o} X_n + S_2^{o} Y_n - Z_n + C_o^{o} \end{bmatrix} \qquad \text{- (1.6a)}$$

where S_1^{o}, S_2^{o} and C_o^{o} are their initial approximate values.

Expressions to be used to perform this parametric least squares adjustment are as follows:-

$$\hat{\delta} = -(A^T PA)^{-1} A^T PW$$

$$\hat{v} = -P^{-1} K \quad \text{for} \quad B = +I$$

$$= P^{-1} K \quad \text{for} \quad B = -I$$

$$\hat{K} = P(A\hat{\delta} + W)$$

$$\hat{X} = X_o + \hat{\delta}; \quad \hat{L} = \hat{L} + \hat{v}$$

Now we can see that there is no difference between (1.2) and (1.3) except that the former uses negative data (Z) and the latter positive. Therefore care must be given to the sign of the depth data used to conform with the chosen mathematical model.

3.0 Determination of the Plane's Parameters

From the least squares adjustment, S_1, S_2 and C_o are determined which consequently gives the equation of the plane. Recalling our knowledge on analytic geometry, the equation of plane in the form of:

$$AX + BY + CZ + D = O \qquad - (1.7)$$

means that A, B and C are the unit vectors of the normal to the plane (Kreyszig, E, 1979). Therefore, in our case here S_1, S_2 and 1 are the unit vectors of the normal of our plane. It is important to stress here, for the sake of clearification, that the normal is perpendicular to the plane and its (normal) direction is away from the plane as shown by fig. 1 below:

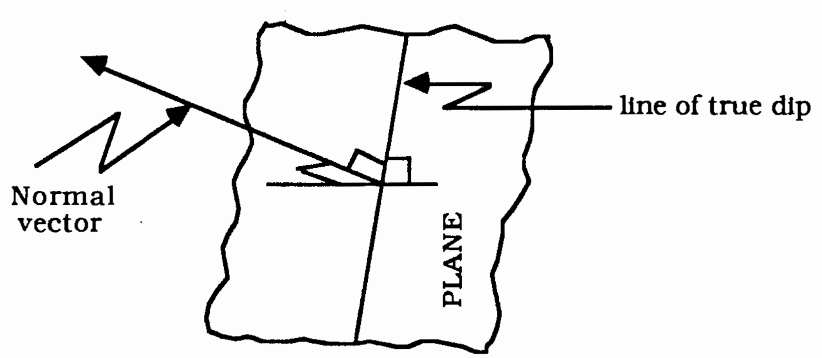

Fig. 1: Plane and its normal

With the above definition together with the values of S_1 and S_2 (also 1 for Z), the following parameters $(\alpha_D\ \alpha_S\ \beta_D)$ of the plane can be determined. It should be noted here that the normal vector goes upward for underground plane and vise-versa

3.1 Azimuth of the dip (α_D)

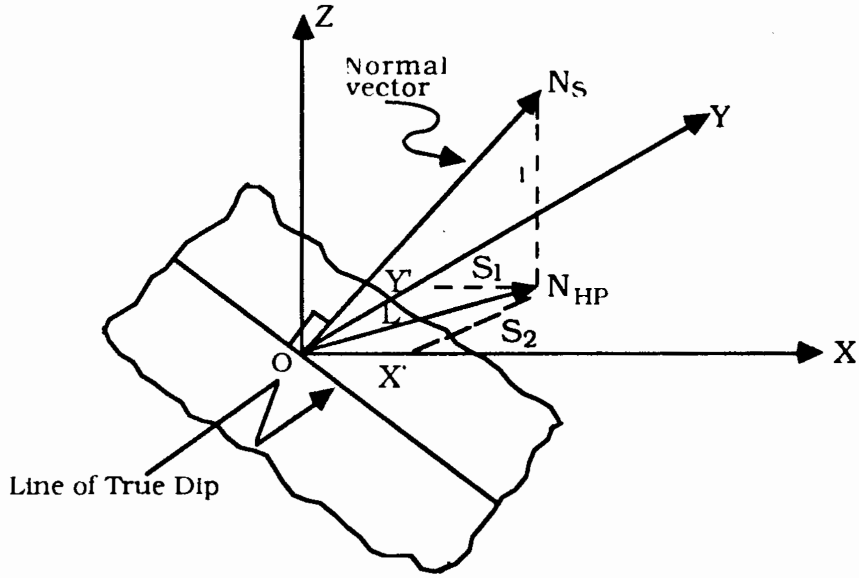

Fig. 2 3D Representation of Normal Vector

Referring to fig. 2 and taking the rectangle $OY^1N_{HP}X^1$ as shown in fig. 3 below,

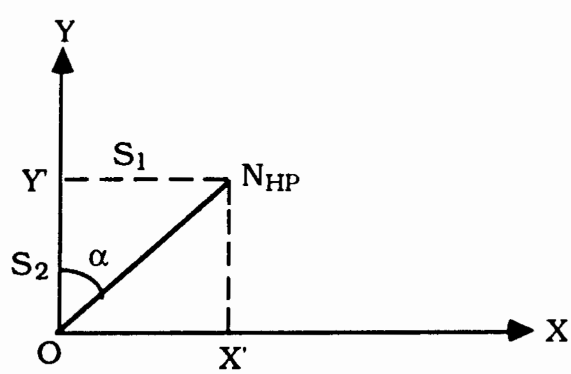

Fig. 3: Determination of Azimuth of Normal

The azimuth of the normal can be determined, from fig. 3, as

$$\alpha_N = \tan^{-1} \frac{S_1}{S_2} \qquad - (1.8)$$

for + S_1 and + S_2

$$\text{or} \quad \alpha_N = 90^\circ + \tan^{-1} \frac{S_2}{S_1} \qquad - (1.9)$$

for + S_1 and - S_2

$$\text{or} \quad \alpha_N = 180^\circ + \tan^{-1} \frac{S_1}{S_2} \qquad - (1.10)$$

for - S_1 and - S_2

$$\text{or} \quad \alpha_N = 270^\circ + \tan^{-1} \frac{S_2}{S_1} \qquad - (1.11)$$

for - S_1 and + S_2

We have determined the azimuth of the normal and were are now looking for the relationship between the azimuths of the normal and the azimuth of the dip. To do so we have to refer to fig. 4 below.

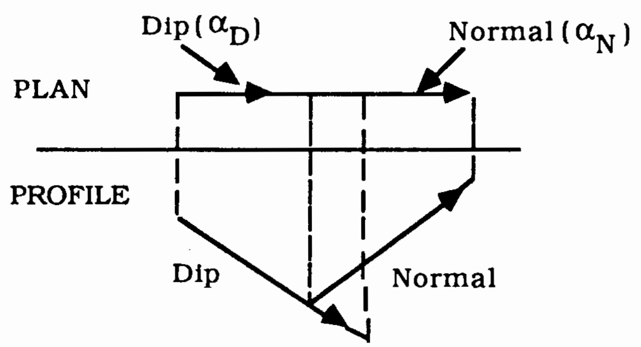

Fig. 4: Relationship Between the Azimuth of the Normal (α_N) and the Azimuth of the Dip (α_D)

Figure 4 shows that both azimuths are equal;

$$\alpha_D = \alpha_N \qquad - (1.12)$$

Proper investigation shows that this is only true for underground plane (- ve Z). In the case of the plane above the earth surface (+ ve Z).

$$\alpha_D = \alpha_N \pm 180^0 \qquad - \quad (1.13)$$

3.2 Azimuth of the Strike (α_S)

By definition, the strike is perpendicular to the line of dip (true dip). Therefore the azimuth of the strike is obtained using the known azimuth of dip as:

$$\alpha_S = \alpha_D \pm 90^0 \qquad - \quad (1.14)$$

3.3 Angle of dip (slope) of the plain (β_D)

In order to determine the angle of dip, we first have to determine the slope of the normal. This can be done by taking a plane section through $ON_S\ N_{HP}$ in fig. 2 and redraw it as shown as fig. 5 below:

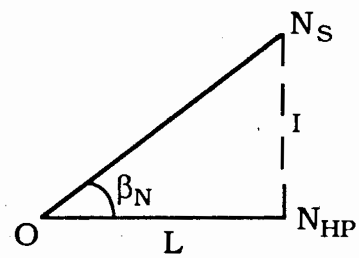

Fig. 5 : Determination of the Slope of theNormal (βN)

From fig. 3

$$L = \sqrt{S_1^2 + S_2^2} \qquad - \quad (1.15)$$

From fig. 5

$$\beta_N = \tan^{-1} \frac{1}{L} \qquad - \quad (1.16)$$

Now, we have to look at the relationship between the angle of dip and slope of the normal by referring to fig. 6 below.

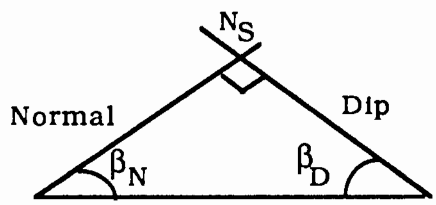

Fig. 6 : Relationship Between the Angle of Dip and the Slope of the Normal

The figure obviously shows that:

$$\beta_D = 90^\circ - \beta_N = \text{Angle of Dip}$$

$$= 90^\circ - \tan^{-1} \frac{1}{\sqrt{S_1^2 + S_2^2}} \qquad - (1.17)$$

4.0 Common Problems

In optimizing the mining operation, it is often necessary to determine the distance from a certain point to the plane at certain slope and azimuth. In this section, step by step procedure to solve this problem will be shown.

4.1 Shortest distance from a point to the plane

If the coordinates of the point P are known, the shortest distance from that point to the plane can be determined using (Kreyszig , E, 1979):

$$DP = \frac{S_1 X_P + S_2 Y_P + Z_P + C}{\sqrt{S_1^2 + S_2^2 + 1^2}} \qquad - (1.18)$$

Naturally this line is parallel but in opposite direction to the normal of the plane.

Therefore:

$$\alpha_{DP} = \alpha_N \pm 180 \qquad - (1.19)$$

$$\beta_{DP} = B_D \qquad - (1.20)$$

Note:

a) $\pm Z$ means that one has to use the proper sign as he uses in the mathematical model of the plane.

b) equation (1.19) is only true if point P is above the plane (+ve D_P). For a point below the plane (-ve D_P), $\alpha_{DP} = \alpha_N$

4.2 Distances from a point to the plane at certain slope and azimuth and also the elevation on the plane where the line meets the plane

This is a common problem in mining surveying. To simplify the matter, step by step solution will be given here.

i) from point P with given slope β_P and azimuth $\alpha_P = \alpha_D + 180^0$ (opposite direction of the dip)

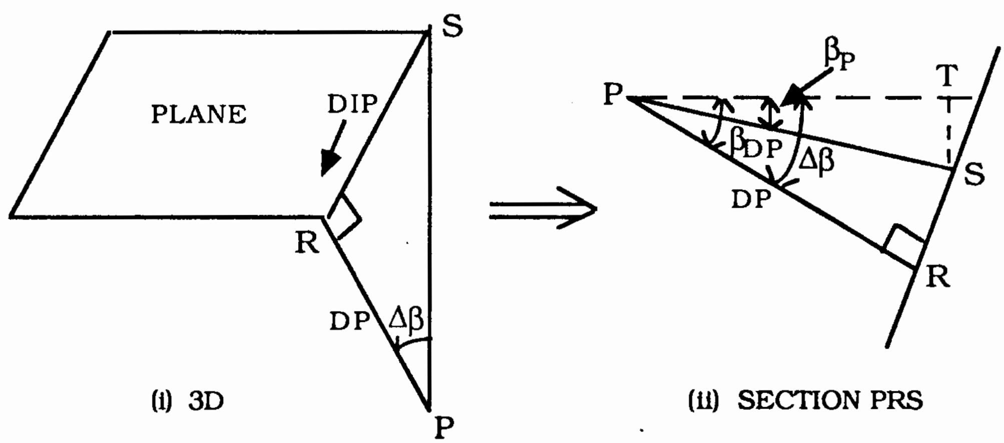

Fig. 7 : Determination of Distance and Elevation

From the above figures and with DP and β_{DP} known (calculated using 1.18 and 1.20) we can determine the required values as follows:

$$\Delta\beta = \beta_{DP} - \beta_P \qquad - (1.21)$$

$$\therefore PS = \frac{DP}{\cos \Delta\beta} \text{ - (slope distance)} \qquad - (1.22)$$

$$\text{and } PT = PS * \cos\beta_P = (DP/\cos \Delta\beta) * \cos\beta_P \text{ (horizontal distance)} \qquad - (1.23)$$

$$ST = PS * \sin\beta_P = (DP/\cos \Delta\beta) * \cos\beta_P \text{ (difference in elevation)} \qquad - (1.24)$$

$\therefore$ Elevation at S = Elevation P + ST

Note: If β_P is upwards, $\Delta\beta = \beta_{DP} + \beta_P$ - (1.25)

ii) From point P with given slope β_P and azimuth α_P..

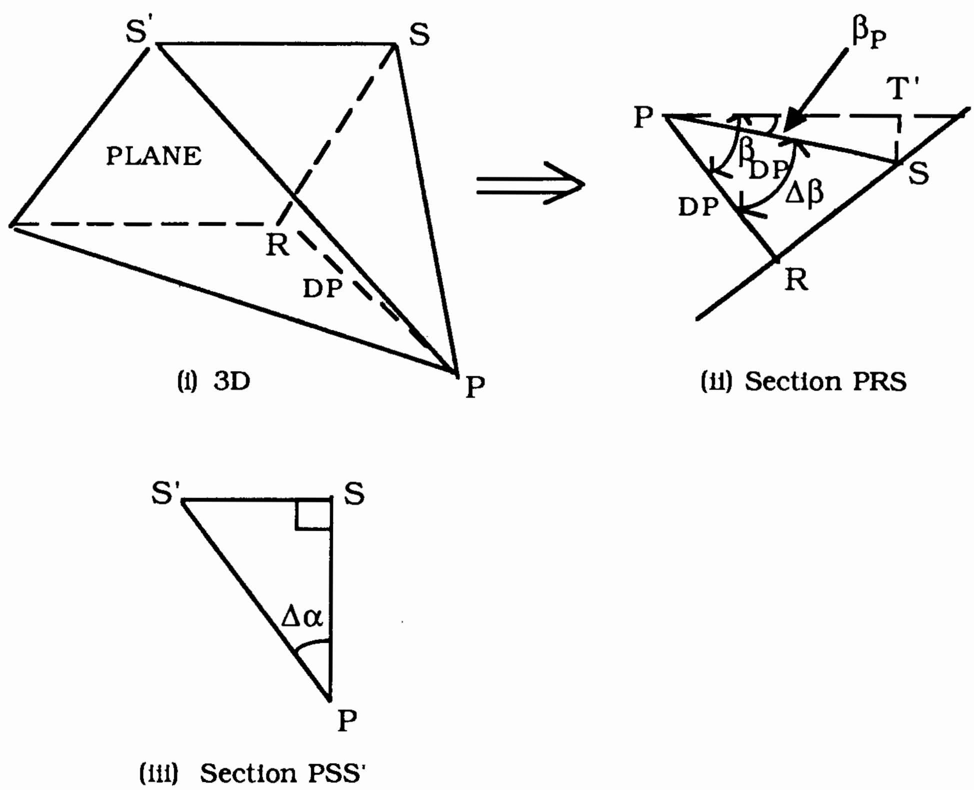

Fig. 8 : Determination of Distance and Elevation for Certain Azimuth and Slope

This is the case where the azimuth of the line, from P to the plane differs from the azimuth (α_{DP}) of the shortest distance DP. In this case the computation is as follow:

a) compute PS as in 4.2 (i) using equations (1.21) and (1.22)

b) compute the difference in azimuth between the line and the line of the shortest distance D_P using

$$\Delta\alpha = \alpha_{DP} - \alpha_{P'} \qquad - (1.26)$$

c) $\therefore PS' = PS * \sin\Delta\alpha$ (slope distance) - (1.27)

$PT = PS' * \sin\Delta\alpha$ (horizontal dist.) - (1.28)

Elevation at S' equal o elevation at S

$$Z_{S'} = Z_S$$
$$= Z_p + ST$$

where ST is computed using e.g. (1.24)

5.0 Numerical Examples

Three numerical examples of the analytical solution developed in this paper are given in appendices as follows:

i) To compute the equation of plane as in section 2, the plane's parameters as in section 3 and the shortest distance from a point to the plane as in section 4.1(App. I).

ii) As in (i) together with the computation of slope and horizontal distance from a point to the plane in the opposite direction of the dip as well as the elevation on the plane where the line strikes the plane and the slope of the line is given (App. II).

iii) As in (ii) but the direction of the line is given (not specifically in opposite direction of the dip) (App. III)

6.0 Conclusion

The method developed here allows the determination of the strike and dip parameters and consequently solving borehole problems analitically. The solution of borehole problems can now be computed by using computers. It needs a writing of a simple computer program where the first part of it involves Least Squares Adjustment to determine the normal vector of the plane of the ore body. The second part of the programe uses the unit vectors determined earlier to solve the borehole problems.

This method as tested in a few examples shown in 5 has been found to be working. it eleminates the use of diagrams and hence can be used universally for all borehole problems with ease and speed.

References

1. Hellen, J.E., Problem and Solutions for Mine Survey (2nd Edition), The Institution of Mine Surveyors, South Africa.

2. Kreyszig, E, 1979 : Advance Engineering Mathmetics, (4th Edition), John Wiley & Sons.

3. Wells, D.E. and Krakiwsky, E.J., 1971 : The Method of Least Squares, Lecture Note No. 18, Dept. of Surveying Engineering, University of New Brunswick.

APPENDIX I

1. Data

Point	Easting (m)	Northing (m)	Depth (m)
1	2000.0	1400.0	540.0
2	2200.0	1400.0	648.0
3	2400.0	1400.0	758.0
4	2600.0	1400.0	866.0
5	2000.0	1200.0	604.0
6	2200.0	1200.0	714.0
7	2400.0	1200.0	821.0
8	2600.0	1200.0	930.0

2. Equation of the Plane

$$S_1 X + S_2 Y - Z + C_0 = 0$$

where

$S_1 = 0.543$, $S_2 = -0.321$, $C_0 = -96.725$

3. Strike and Dip Parameters

a)	Azimuth of the Dip	=	120° 35' 52".4
b)	Azimuth of the Strike	=	30° 35' 52".4
c)	Dip Angle	=	32° 15' 25".8

4. Shortest Distance From a Point to the Plane

a) Coordinates of the Point X

$X = 2200.000$ m
$Y = 800.000$ m
$Z = 627.000$ m

b) i) Shortest Distance = 181.331 m
ii) Slope = 57° 44' 34".2

5. Distance from Point X to the Plane at a given Slope (0° 00' 00")

a)	Azimuth	=	300° 35' 52".4
b)	Slope Distance	=	339.749 m
c)	Horizontal Distance	=	339.749 m
d)	Elevation on Plane	=	627.000 m

APPENDIX II

1. Data

Point	Easting (m)	Northing (m)	Depth (m)
1	10000.0	10000.0	1000.0
2	9463.8	10450.0	1100.0
3	8900.0	8094.7	1200.0
4	8900.0	8094.2	1200.2

2. Equation of the Plane

$$S_1 X + S_2 Y - Z + C_o = 0$$

where

$S_1 = -0.185$, $S_2 = -0.002$, $S_o = 2832.426$

3. Azimuth of the Dip and the Strike

a) Azimuth of the Dip = $270^o\ 32'\ 46''.3$
b) Azimuth of the Strike = $180^o\ 32'\ 46''.3$
c) Dip Angle = $10^o\ 28'\ 55''.3$

4. Distances from A Point (X) to the Plane

a) Shortest Distance = 1121.034m with
Slope = $79^o\ 31'\ 04''.7$

b) The Distance from Pocat X ata a given slope of $15^o\ 00'\ 00''$: Azimuth of $90^o\ 32'\ 46''.3$

i) Slope Distance = 2605.673 m
ii) Horizontal Distance = 2516.887 m
iii) Elevation on Plane = 1274.398 m

Note : Coordinates of X are:

X = 6000.182 m
Y = 10038.132 m
Z = 600.000 m

APPENDIX III

1. Data

Point	Easting (m)	Northing (m)	Depth (m)
1	10866.0	9500.0	700.0
2	9568.0	9060.3	1000.0
3	10223.2	8734.0	1200.0
4	10223.0	8734.0	1200.0

2. Equation of the Plane

$$S_1 X + S_2 Y - Z + C_o = 0$$

where

$S_1 = -0.015$, $S_2 = -0.640$, $C_o = 6946.740$

3. Azimuth of the dip and the Strike

a) Azimuth of the Dip = $181^o\ 22'\ 52''.3$
b) Azimuth of the Strike = $91^o\ 22'\ 52''.3$
c) Dip Angle = $32^o\ 37'\ 22''.5$

4. Distances from Point (X) to the Plane

a) Shortest Distance = - 510.893 m
Slope = $57^o\ 22'\ 37''.5$

b) Distances at a given Slope and Azimuth

i) Slope = $0^o\ 34'\ 23''.0$; Azimuth = $181^o\ 22'\ 52''.3$
Slope Distance = - 933.130 m
Horizontal Distance = - 933.084 m
Elevation of Plane = 990.667 m

ii) Slope = $0^o\ 34'\ 23''.0$; Azimuth = $160^o\ 00'\ 000''$

Note:

a) Coordinates of X are:

X = 10000.000
Y = 10000.000
Z = 1000.000

b) -Ve distances above indicate that point X is below the plane.